UNIT 5

Measurement Geometry

MODULE 11

Angle Relationships in Parallel Lines and Triangles

FL 8.G.1.5

MODULE 12

The Pythagorean Theorem

FL 8.G.2.6, 8.G.2.7, 8.G.2.8

MODULE 13

Volume

FL 8.G.3.9

CAREERS IN MATH

Hydrologist A hydrologist is a scientist who studies and solves water-related issues. A hydrologist might work to prevent or clean up polluted water sources, locate water supplies for urban or rural needs, or control flooding and erosion. A hydrologist uses math to assess water resources and mathematical models to understand water systems, as well as statistics to analyze phenomena such as rainfall patterns. If you are interested in a career as a hydrologist, you should study the following mathematical subjects:

- Algebra
- Trigonometry
- Calculus
- Statistics

Research other careers that require creating and using mathematical models to understand physical phenomena.

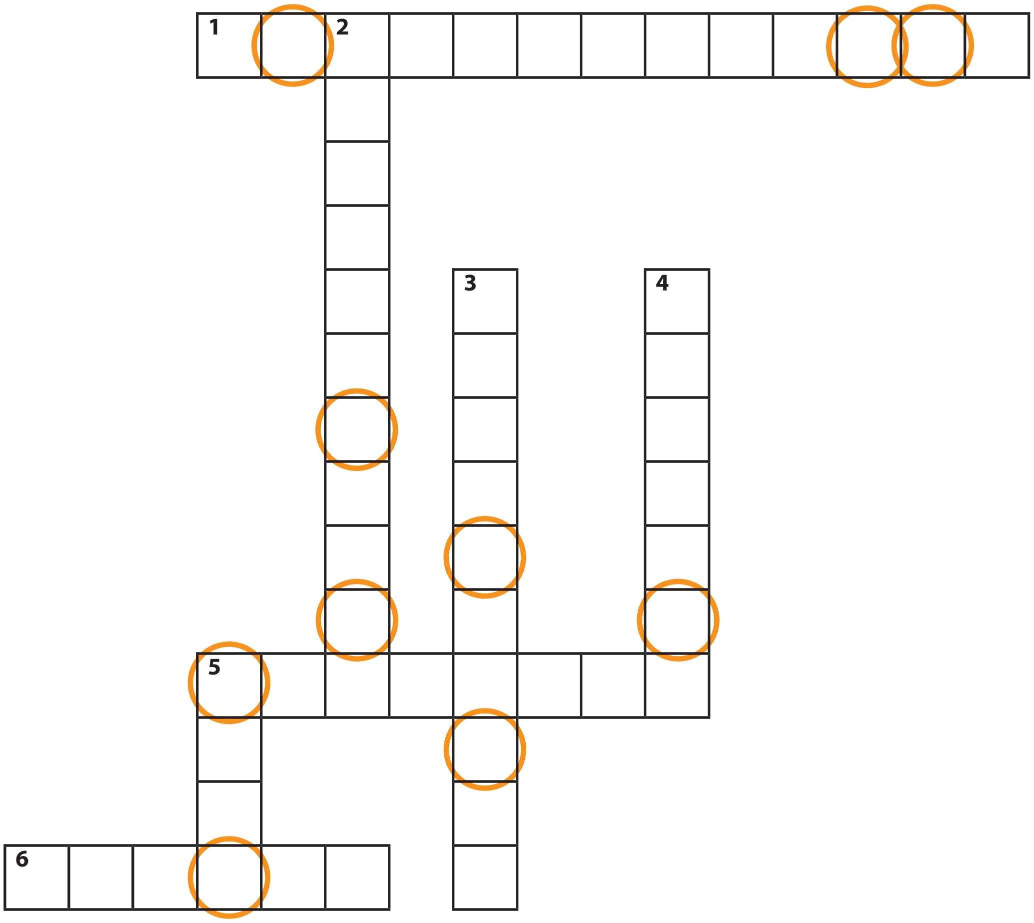
Unit 5 Performance Task

At the end of the unit, check out how **hydrologists** use math.

UNIT 5

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.



Across

1. The angle formed by two sides of a triangle (2 words) (Lesson 11.2)
5. A three-dimensional figure that has two congruent circular bases. (Lesson 13.1)
6. A three-dimensional figure with all points the same distance from the center. (Lesson 13.3)

Down

2. The line that intersects two or more lines. (Lesson 11.1)
3. The side opposite the right angle in a right triangle. (Lesson 12.1)
4. Figures with the same shape but not necessarily the same size. (Lesson 11.3)
5. A three-dimensional figure that has one vertex and one circular base. (Lesson 13.2)

Q: What do you call an angle that is adorable?

A: ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Angle Relationships in Parallel Lines and Triangles

MODULE 11

ESSENTIAL QUESTION

How can you use angle relationships in parallel lines and triangles to solve real-world problems?

LESSON 11.1

Parallel Lines Cut by a Transversal

FL 8.G.1.5

LESSON 11.2

Angle Theorems for Triangles

FL 8.EE.3.7, 8.EE.3.7b, 8.G.1.5

LESSON 11.3

Angle-Angle Similarity

FL 8.EE.2.6, 8.EE.3.7, 8.G.1.5

Real-World Video

Many cities are designed on a grid with parallel streets. If another street runs across the parallel lines, it is a transversal. Special relationships exist between parallel lines and transversals.

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Complete these exercises to review skills you will need for this module.

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Solve Two-Step Equations

EXAMPLE

$7x + 9 = 30$	Write the equation.
$7x + 9 - 9 = 30 - 9$	Subtract 9 from both sides.
$7x = 21$	Simplify.
$\frac{7x}{7} = \frac{21}{7}$	Divide both sides by 7.
$x = 3$	Simplify.

Solve for *x*.

1. $6x + 10 = 46$ ________

2. $7x - 6 = 36$ ________

3. $3x + 26 = 59$ ________

4. $2x + 5 = -25$ ________

5. $6x - 7 = 41$ ________

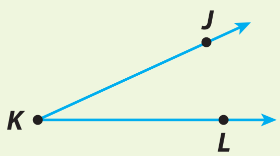
6. $\frac{1}{2}x + 9 = 30$ ________

7. $\frac{1}{3}x - 7 = 15$ ________

8. $0.5x - 0.6 = 8.4$ ________

Name Angles

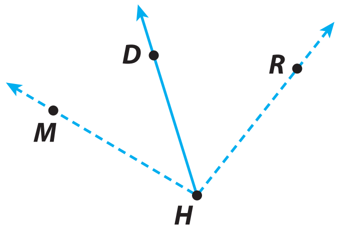
EXAMPLE



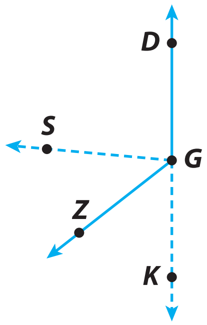
Use three points of an angle, including the vertex, to name the angle. Write the vertex between the other two points: $\angle JKL$ or $\angle LKJ$. You can also use just the vertex letter to name the angle if there is no danger of confusing the angle with another. This is also $\angle K$.

Give two names for the angle formed by the dashed rays.

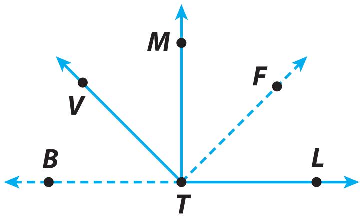
9. ________



10. ________



11. ________

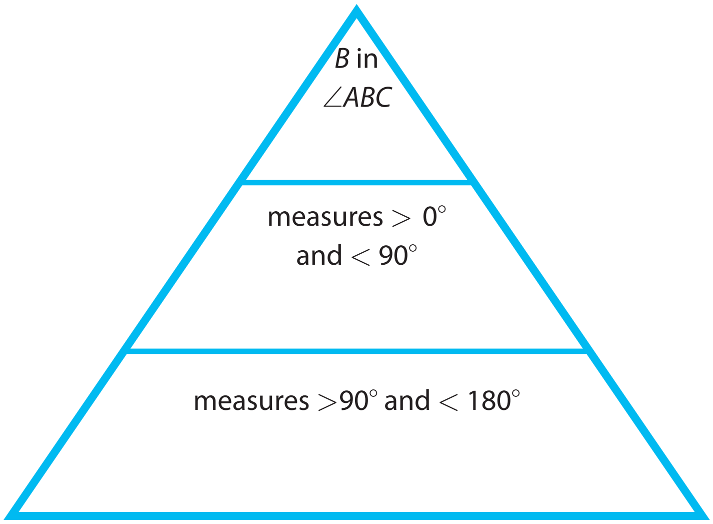


Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You can put more than one word in each section of the triangle.

Reviewing Angles



Understand Vocabulary

Complete the sentences using preview words.

1. A line that intersects two or more lines is a ____________________.
2. Figures with the same shape but not necessarily the same size are ____________________.
3. An ____________________ is an angle formed by one side of the triangle and the extension of an adjacent side.

Vocabulary

Review Words

- ✔ acute angle *(ángulo agudo)*
- ✔ angle *(ángulo)*
- congruent *(congruente)*
- ✔ obtuse angle *(ángulo obtuso)*
- parallel lines *(líneas paralelas)*
- ✔ vertex *(vértice)*

Preview Words

- alternate exterior angles *(ángulos alternos externos)*
- alternate interior angles *(ángulos alternos internos)*
- corresponding angles *(ángulos correspondientes (para líneas))*
- exterior angle *(ángulo externo de un polígono)*
- interior angle *(ángulos internos)*
- remote interior angle *(ángulo interno remoto)*
- same-side interior angles *(ángulos internos del mismo lado)*
- similar *(semejantes)*
- transversal *(transversal)*

Active Reading

Pyramid Before beginning the module, create a pyramid to help you organize what you learn. Label each side with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas on the appropriate side.

MODULE 11

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.G.1.5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Key Vocabulary

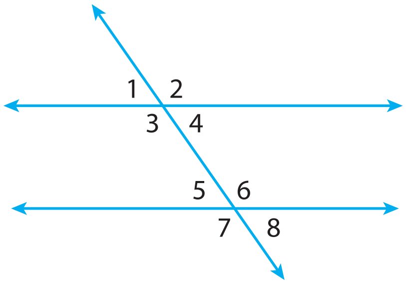
transversal *(transversal)*
A line that intersects two or more lines.

What It Means to You

You will learn about the special angle relationships formed when parallel lines are intersected by a third line called a transversal.

UNPACKING EXAMPLE 8.G.1.5

Which angles formed by the transversal and the parallel lines seem to be congruent?



It appears that the angles below are congruent.

$\angle 1 \cong \angle 4 \cong \angle 5 \cong \angle 8$

$\angle 2 \cong \angle 3 \cong \angle 6 \cong \angle 7$

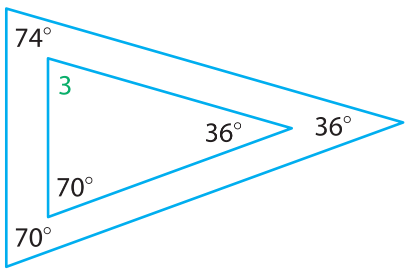
FL 8.G.1.5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

What It Means to You

You will use the angle-angle criterion to determine similarity of two triangles.

UNPACKING EXAMPLE 8.G.1.5



Explain whether the triangles are similar.

Two angles in the large triangle are congruent to two angles in the smaller triangle, so the third pair of angles must also be congruent, which makes the triangles similar.

$70° + 36° + m\angle 3 = 180°$

$m\angle 3 = 74°$

Visit **my.hrw.com** to see all **Florida Math Standards** unpacked.

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LESSON 11.1

Parallel Lines Cut by a Transversal

FL 8.G.1.5

Use informal arguments to establish facts about… the angles created when parallel lines are cut by a transversal.…

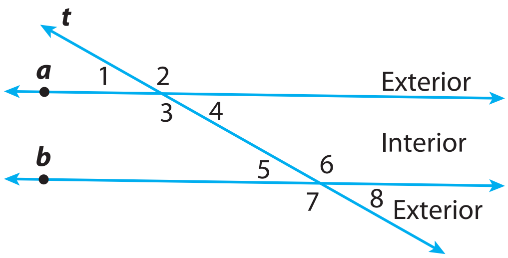
ESSENTIAL QUESTION

What can you conclude about the angles formed by parallel lines that are cut by a transversal?

EXPLORE ACTIVITY 1 FL 8.G.1.5

Parallel Lines and Transversals

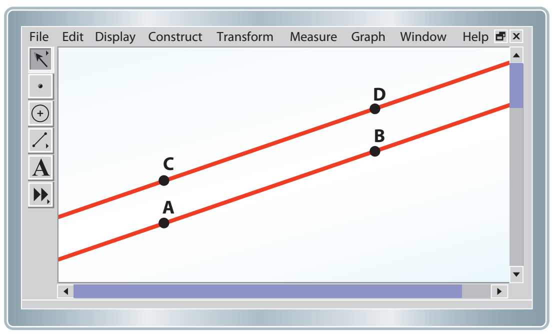
A **transversal** is a line that intersects two lines in the same plane at two different points. Transversal *t* and lines *a* and *b* form eight angles.



Angle Pairs Formed by a Transversal

Term	Example
Corresponding angles lie on the same side of the transversal *t*, on the same side of lines *a* and *b*.	$\angle 1$ and $\angle 5$
Alternate interior angles are nonadjacent angles that lie on opposite sides of the transversal *t*, between lines *a* and *b*.	$\angle 3$ and $\angle 6$
Alternate exterior angles lie on opposite sides of the transversal *t*, outside lines *a* and *b*.	$\angle 1$ and $\angle 8$
Same-side interior angles lie on the same side of the transversal *t*, between lines *a* and *b*.	$\angle 3$ and $\angle 5$

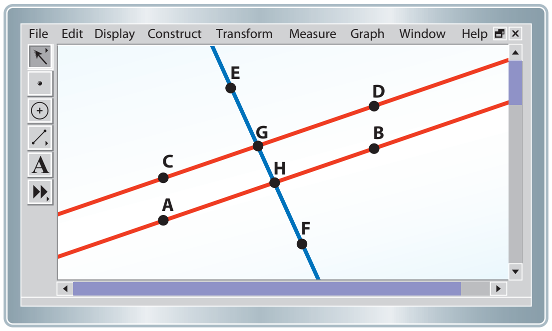
Use geometry software to explore the angles formed when a transversal intersects parallel lines.



A Construct a line and label two points on the line *A* and *B*.

B Create point *C* not on $\overleftrightarrow{AB}$. Then construct a line parallel to $\overleftrightarrow{AB}$ through point *C*. Create another point on this line and label it *D*.

C Create two points outside the two parallel lines and label them E and F. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection G and H.



D Measure the angles formed by the parallel lines and the transversal. Write the angle measures in the table below.

E Drag point E or point F to a different position. Record the new angle measures in the table.

Angle	$\angle CGE$	$\angle DGE$	$\angle CGH$	$\angle DGH$	$\angle AHG$	$\angle BHG$	$\angle AHF$	$\angle BHF$
Measure								
Measure								

Reflect

Make a Conjecture **Identify the pairs of angles in the diagram. Then make a conjecture about their angle measures. Drag a point in the diagram to confirm your conjecture.**

1. corresponding angles

______

______

______

2. alternate interior angles

______

______

3. alternate exterior angles

______

______

4. same-side interior angles

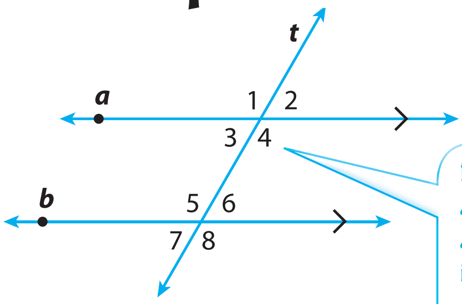
______

______

EXPLORE ACTIVITY 2 FL 8.G.1.5

Justifying Angle Relationships

You can use tracing paper to informally justify your conclusions from the first Explore Activity.



Recall that vertical angles are the opposite angles formed by two intersecting lines. ∠1 and ∠4 are vertical angles.

Lines *a* and *b* are parallel. (The black arrows on the diagram indicate parallel lines.)

A Trace the diagram onto tracing paper.

B Position the tracing paper over the original diagram so that ∠1 on the tracing is over ∠5 on the original diagram. Compare the two angles. Do they appear to be congruent?

____

____

Math Talk
Mathematical Practices
What do you notice about the special angle pairs formed by the transversal?

C Use the tracing paper to compare all eight angles in the diagram to each other. List all of the congruent angle pairs.

____

____

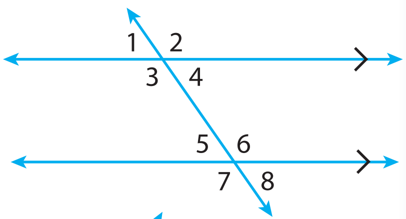
____

Finding Unknown Angle Measures

You can find any unknown angle measure when two parallel lines are cut by a transversal if you are given at least one other angle measure.

EXAMPLE 1 FL 8.G.1.5

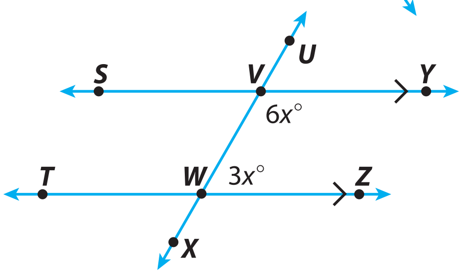
A **Find m∠2 when m∠7 = 125°.**



∠2 is congruent to ∠7 because they are alternate exterior angles.

Therefore, m∠2 = 125°.

B **Find m∠*VWZ*.**



∠*VWZ* is supplementary to ∠*YVW* because they are same-side interior angles.
m∠*VWZ* + m∠*YVW* = 180°

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From the previous page, $m\angle VWZ + m\angle YVW = 180°$, $m\angle VWZ = 3x°$, and $m\angle YVW = 6x°$.

$m\angle VWZ + m\angle YVW = 180°$

$3x° + 6x° = 180°$ Replace $m\angle VWZ$ with $3x°$ and $m\angle YVW$ with $6x°$.

$9x = 180$ Combine like terms.

$\frac{9x}{9} = \frac{180}{9}$ Divide both sides by 9.

$x = 20$ Simplify.

$m\angle VWZ = 3x° = (3 \cdot 20)° = 60°$

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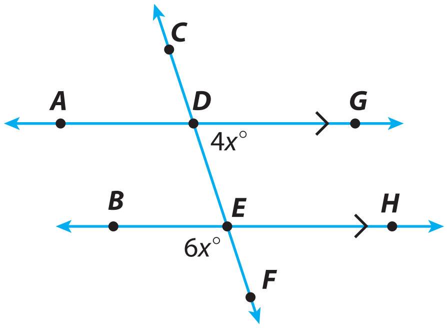
YOUR TURN

Find each angle measure.



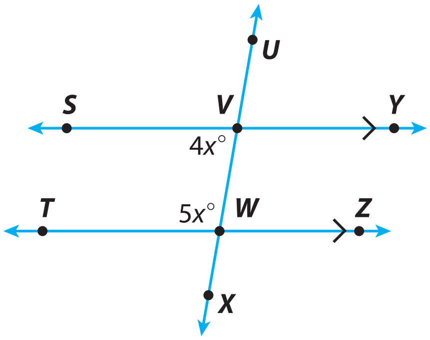
5. $m\angle GDE =$ ____________

6. $m\angle BEF =$ ____________

7. $m\angle CDG =$ ____________

Guided Practice

Use the figure for Exercises 1–4. (Explore Activity 1 and Example 1)



1. $\angle UVY$ and ________ are a pair of corresponding angles.

2. $\angle WVY$ and $\angle VWT$ are ____________________ angles.

3. Find $m\angle SVW$. ____________

4. Find $m\angle VWT$. ____________

5. **Vocabulary** When two parallel lines are cut by a transversal, ____________________ angles are supplementary. (Explore Activity 1)

ESSENTIAL QUESTION CHECK-IN

6. What can you conclude about the interior angles formed when two parallel lines are cut by a transversal?

____

____

Name ______________________ Class ______________ Date ______________

11.1 Independent Practice

FL 8.G.1.5

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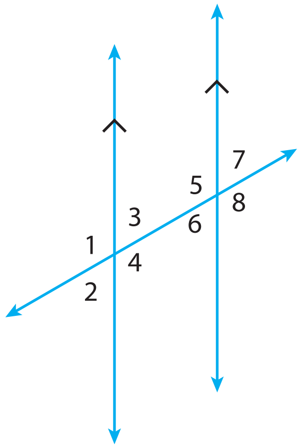
Vocabulary **Use the figure for Exercises 7–10.**



7. Name all pairs of corresponding angles.

____

8. Name both pairs of alternate exterior angles.

____

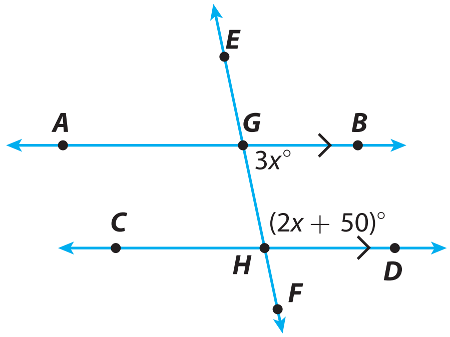
9. Name the relationship between $\angle 3$ and $\angle 6$.

____

10. Name the relationship between $\angle 4$ and $\angle 6$.

____

Find each angle measure.



11. $m\angle AGE$ when $m\angle FHD = 30°$ __________

12. $m\angle AGH$ when $m\angle CHF = 150°$ __________

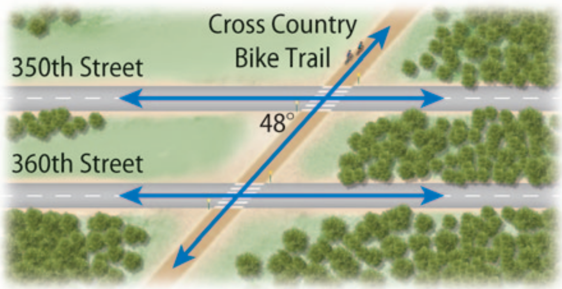
13. $m\angle CHF$ when $m\angle BGE = 110°$ __________

14. $m\angle CHG$ when $m\angle HGA = 120°$ __________

15. $m\angle BGH =$ __________

16. $m\angle GHD =$ __________

17. The Cross Country Bike Trail follows a straight line where it crosses 350th and 360th Streets. The two streets are parallel to each other. What is the measure of the larger angle formed at the intersection of the bike trail and 360th Street? Explain.



____

____

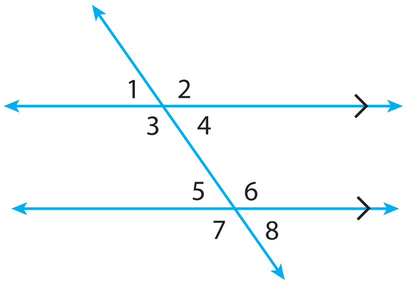
____

18. **Critical Thinking** How many different angles would be formed by a transversal intersecting three parallel lines? How many different angle measures would there be?

____

____

19. **Communicate Mathematical Ideas** In the diagram at the right, suppose m∠6 = 125°. Explain how to find the measures of each of the other seven numbered angles.



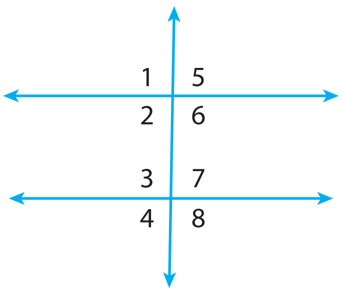
H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

20. **Draw Conclusions** In a diagram showing two parallel lines cut by a transversal, the measures of two same-side interior angles are both given as $3x°$. Without writing and solving an equation, can you determine the measures of both angles? Explain. Then write and solve an equation to find the measures.

21. **Make a Conjecture** Draw two parallel lines and a transversal. Choose one of the eight angles that are formed. How many of the other seven angles are congruent to the angle you selected? How many of the other seven angles are supplementary to your angle? Will your answer change if you select a different angle?

22. **Critique Reasoning** In the diagram at the right, ∠2, ∠3, ∠5, and ∠8 are all congruent, and ∠1, ∠4, ∠6, and ∠7 are all congruent. Aiden says that this is enough information to conclude that the diagram shows two parallel lines cut by a transversal. Is he correct? Justify your answer.



LESSON 11.2

Angle Theorems for Triangles

FL 8.G.1.5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles. . . . *Also 8.EE.3.7, 8.EE.3.7b*

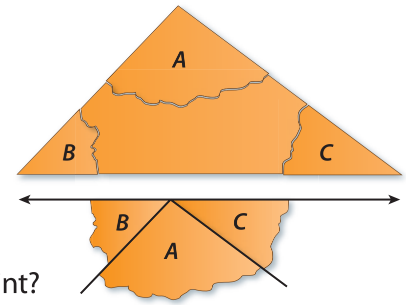
ESSENTIAL QUESTION

What can you conclude about the measures of the angles of a triangle?

EXPLORE ACTIVITY 1 FL 8.G.1.5

Sum of the Angle Measures in a Triangle

There is a special relationship between the measures of the interior angles of a triangle.



A Draw a triangle and cut it out. Label the angles *A*, *B*, and *C*.

B Tear off each "corner" of the triangle. Each corner includes the vertex of one angle of the triangle.

C Arrange the vertices of the triangle around a point so that none of your corners overlap and there are no gaps between them.

D What do you notice about how the angles fit together around a point?

____

E What is the measure of a straight angle? ____________________

F Describe the relationship among the measures of the angles of $\triangle ABC$.

____

The Triangle Sum Theorem states that for $\triangle ABC$, $m\angle A + m\angle B + m\angle C =$ ______.

Reflect

1. **Justify Reasoning** Can a triangle have two right angles? Explain.

____

____

____

2. **Analyze Relationships** Describe the relationship between the two acute angles in a right triangle. Explain your reasoning.

____

____

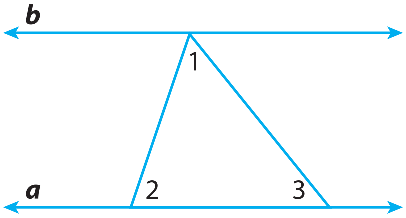
____

EXPLORE ACTIVITY 2 FL 8.G.1.5

Justifying the Triangle Sum Theorem

You can use your knowledge of parallel lines intersected by a transversal to informally justify the Triangle Sum Theorem.

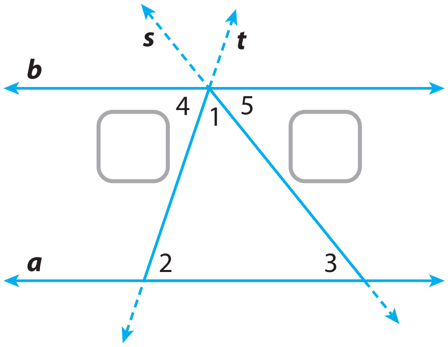
Follow the steps to informally prove the Triangle Sum Theorem. You should draw each step on your own paper. The figures below are provided for you to check your work.



A Draw a triangle and label the angles as ∠1, ∠2, and ∠3 as shown.

B Draw line *a* through the base of the triangle.

C The Parallel Postulate states that through a point not on a line ℓ, there is exactly one line parallel to line ℓ. Draw line *b* parallel to line *a,* through the vertex opposite the base of the triangle.



D Extend each of the non-base sides of the triangle to form transversal *s* and transversal *t*. Transversals *s* and *t* intersect parallel lines *a* and *b*.

E Label the angles formed by line *b* and the transversals as ∠4 and ∠5.

F Because ∠4 and ________________ are alternate interior angles, they are ________________.

Label ∠4 with the number of the angle to which it is congruent.

G Because ∠5 and ________________ are alternate interior angles, they are ________________.

Label ∠5 with the number of the angle to which it is congruent.

H The three angles that lie along line *b* at the vertex of the triangle are ∠1, ∠4, and ∠5. Notice that these three angles lie along a line.

So, m∠1 + m∠2 + m∠5 = ________________.

Because angles 2 and 4 are congruent and angles 3 and 5 are congruent, you can substitute m∠2 for m∠4 and m∠3 for m∠5 in the equation above.

So, m∠1 + m∠2 + m∠3 = ________________.

This shows that the sum of the angle measures in a triangle is always ________________.

Reflect

3. **Analyze Relationships** How can you use the fact that $m\angle 4 + m\angle 1 + m\angle 5 = 180°$ to show that $m\angle 2 + m\angle 1 + m\angle 3 = 180°$?

____

____

____

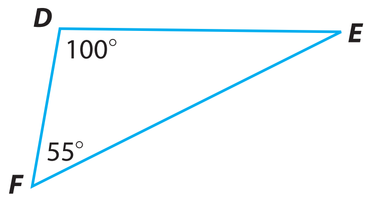
Finding Missing Angle Measures in Triangles

If you know the measures of two angles in a triangle, you can use the Triangle Sum Theorem to find the measure of the third angle.

EXAMPLE 1

FL 8.EE.3.7

Find the missing angle measure.



STEP 1 Write the Triangle Sum Theorem for this triangle.

$m\angle D + m\angle E + m\angle F = 180°$

STEP 2 Substitute the given angle measures.

$55° + m\angle E + 100° = 180°$

STEP 3 Solve the equation for $m\angle E$.

$$\begin{aligned} 55° + m\angle E + 100° &= 180° \\ 155° + m\angle E &= 180° \\ -155° \qquad\quad &\quad -155° \\ m\angle E &= 25° \end{aligned}$$

Simplify.

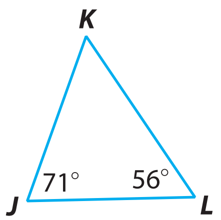
Subtract 155° from both sides.

So, $m\angle E = 25°$.

YOUR TURN

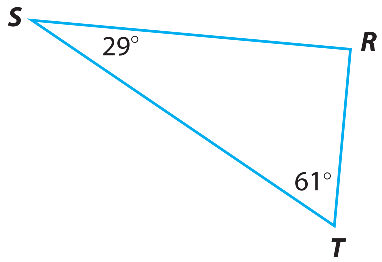
Find the missing angle measure.

4.



$m\angle K =$ ______________

5.



$m\angle R =$ ______________

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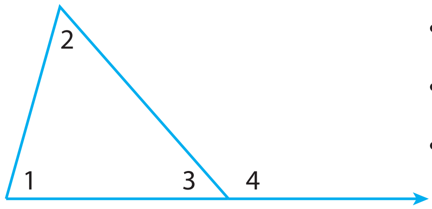
EXPLORE ACTIVITY 3 FL 8.G.1.5

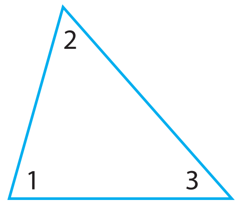
Exterior Angles and Remote Interior Angles

An **interior angle** of a triangle is formed by two sides of the triangle. An **exterior angle** is formed by one side of the triangle and the extension of an adjacent side. Each exterior angle has two remote interior angles. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



- $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles.
- $\angle 4$ is an exterior angle.
- $\angle 1$ and $\angle 2$ are remote interior angles to $\angle 4$.

There is a special relationship between the measure of an exterior angle and the measures of its remote interior angles.



A Extend the base of the triangle and label the exterior angle as $\angle 4$.

B The Triangle Sum Theorem states:

$m\angle 1 + m\angle 2 + m\angle 3 =$ __________.

C $\angle 3$ and $\angle 4$ form a ______________________________,

so $m\angle 3 + m\angle 4 =$ ______________________________.

D Use the equations in **B** and **C** to complete the following equation:

$m\angle 1 + m\angle 2 +$ ______________ $=$ ______________ $+ m\angle 4$

E Use properties of equality to simplify the equation in **D**:

____

The Exterior Angle Theorem states that the measure of an ______________ angle is equal to the sum of its ________________ angles.

Reflect

6. Sketch a triangle and draw all of its exterior angles. How many exterior angles does a triangle have at each vertex?

____

7. How many total exterior angles does a triangle have?

____

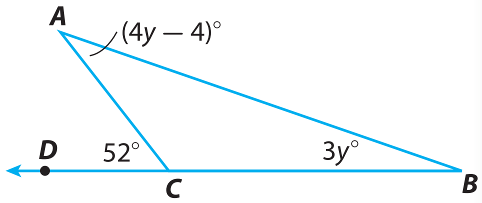
Using the Exterior Angle Theorem

You can use the Exterior Angle Theorem to find the measures of the interior angles of a triangle.

EXAMPLE 2

FL 8.EE.3.7b

Find m∠*A* and m∠*B*.



STEP 1 Write the Exterior Angle Theorem as it applies to this triangle.

$m\angle A + m\angle B = m\angle ACD$

STEP 2 Substitute the given angle measures.

$(4y - 4)° + 3y° = 52°$

STEP 3 Solve the equation for *y*.

$(4y - 4)° + 3y° = 52°$

$4y° - 4° + 3y° = 52°$ Remove parentheses.

$7y° - 4° = 52°$ Simplify.

$+4° \quad +4°$ Add 4° to both sides.

$7y° = 56°$ Simplify.

$\frac{7y°}{7} = \frac{56°}{7}$ Divide both sides by 7.

$y = 8$ Simplify.

STEP 4 Use the value of *y* to find m∠*A* and m∠*B*.

$$\begin{aligned} m\angle A &= 4y - 4 \\ &= 4(8) - 4 \\ &= 32 - 4 \\ &= 28 \end{aligned} \qquad \begin{aligned} m\angle B &= 3y \\ &= 3(8) \\ &= 24 \end{aligned}$$

So, $m\angle A = 28°$ and $m\angle B = 24°$.

Math Talk
Mathematical Practices

Describe two ways to find m∠*ACB*.

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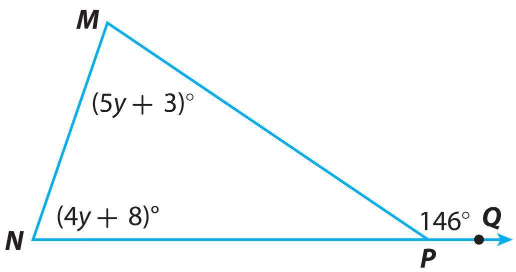
YOUR TURN

8. Find m∠*M* and m∠*N*.

m∠*M* = ___________

m∠*N* = ___________



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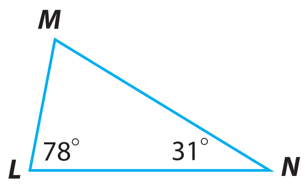
Guided Practice

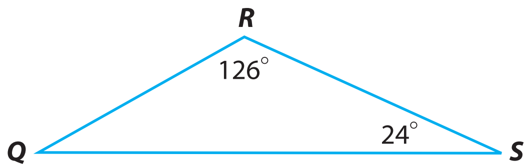
Find each missing angle measure. (Explore Activity 1 and Example 1)

1.



$m\angle M =$ ________________

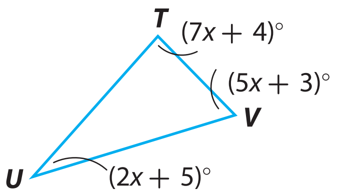
2.



$m\angle Q =$ ________________

Use the Triangle Sum Theorem to find the measure of each angle in degrees. (Explore Activity 2 and Example 1)

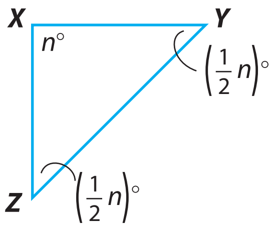
3.



$m\angle T =$ ________, $m\angle U =$ ________,

$m\angle V =$ ________

4.

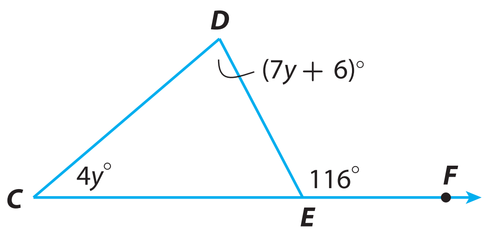


$m\angle X =$ ________, $m\angle Y =$ ________,

$m\angle Z =$ ________

Use the Exterior Angle Theorem to find the measure of each angle in degrees. (Explore Activity 3 and Example 2)

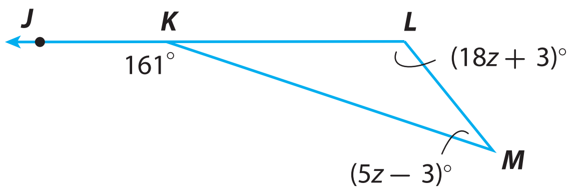
5.



$m\angle C =$ ________, $m\angle D =$ ________,

$m\angle DEC =$ ________

6.



$m\angle L =$ ________, $m\angle M =$ ________,

$m\angle LKM =$ ________

ESSENTIAL QUESTION CHECK-IN

7. Describe the relationships among the measures of the angles of a triangle.

____

____

____

Name ______________________ Class ______________ Date ____________

11.2 Independent Practice

FL 8.EE.3.7, 8.EE.3.7b, 8.G.1.5

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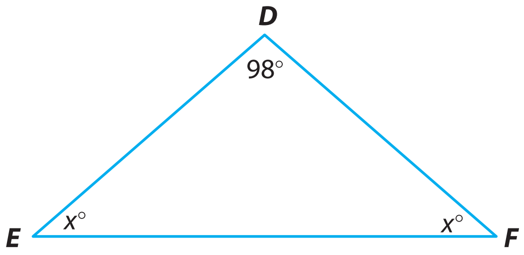
Find the measure of each angle.

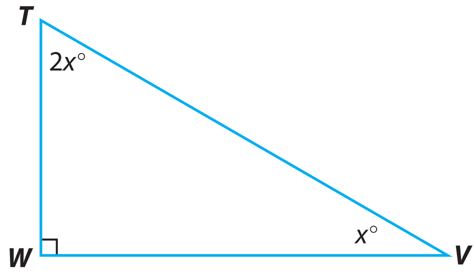
8.



m∠E = ______________

m∠F = ______________

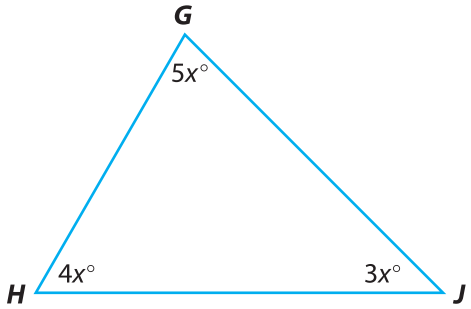
9.



m∠T = ______________

m∠V = ______________

10.

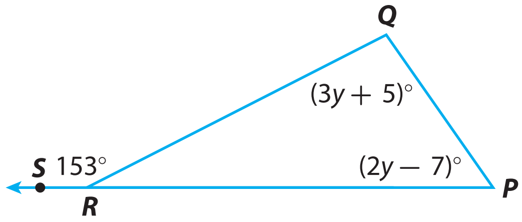


m∠G = ______________

m∠H = ______________

m∠J = ______________

11.

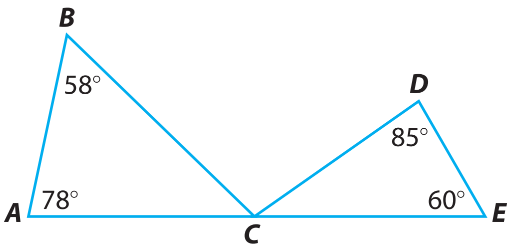


m∠Q = ______________

m∠P = ______________

m∠QRP = ______________

12.

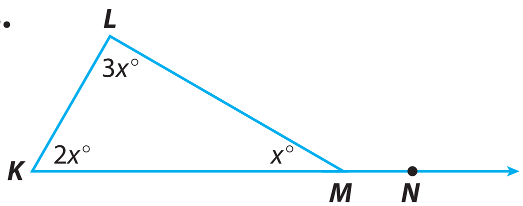


m∠ACB = ______________

m∠BCD = ______________

m∠DCE = ______________

13.



m∠K = ______________

m∠L = ______________

m∠KML = ______________

m∠LMN = ______________

14. **Multistep** The second angle in a triangle is five times as large as the first. The third angle is two-thirds as large as the first. Find the angle measures. ______________

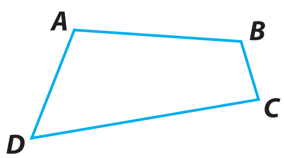
15. **Analyze Relationships** Can a triangle have two obtuse angles? Explain.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. **Critical Thinking** Explain how you can use the Triangle Sum Theorem to find the measures of the angles of an equilateral triangle.

17. **a.** **Draw Conclusions** Find the sum of the measures of the angles in quadrilateral *ABCD*. (Hint: Draw diagonal $\overline{AC}$. How can you use the figures you have formed to find the sum?)



Sum = ____________

b. **Make a Conjecture** Write a "Quadrilateral Sum Theorem." Explain why you think it is true.

18. **Communicate Mathematical Ideas** Describe two ways that an exterior angle of a triangle is related to one or more of the interior angles.

LESSON 11.3

Angle-Angle Similarity

FL 8.G.1.5

Use informal arguments to establish facts about … the angle-angle criterion for similarity of triangles. *Also 8.EE.2.6, 8.EE.3.7*

ESSENTIAL QUESTION

How can you determine when two triangles are similar?

EXPLORE ACTIVITY 1 FL 8.G.1.5

Discovering Angle-Angle Similarity

Similar figures have the same shape but may have different sizes. Two triangles are **similar** if their corresponding angles are congruent and the lengths of their corresponding sides are proportional.

A Use your protractor and a straightedge to draw a triangle. Make one angle measure 45° and another angle measure 60°.

B Compare your triangle to those drawn by your classmates. How are the triangles the same?

____

How are they different?

____

C Use the Triangle Sum Theorem to find the measure of the third angle of your triangle.

____

Reflect

1. If two angles in one triangle are congruent to two angles in another triangle, what do you know about the third pair of angles?

____

2. **Make a Conjecture** Are two pairs of congruent angles enough information to conclude that two triangles are similar? Explain.

____

____

____

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Using the AA Similarity Postulate

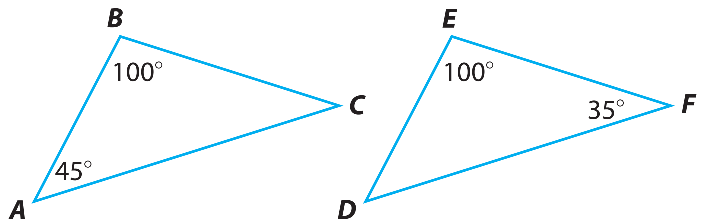
Angle-Angle (AA) Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

EXAMPLE 1

FL 8.G.1.5

Explain whether the triangles are similar.



The figure shows only one pair of congruent angles. Find the measure of the third angle in each triangle.

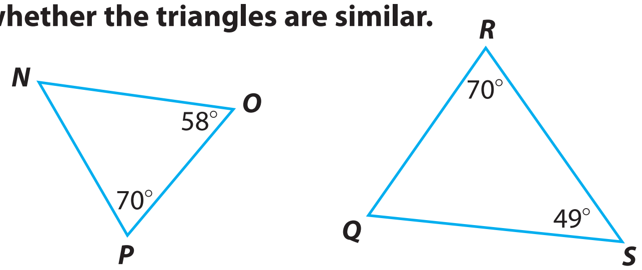
$45° + 100° + m\angle C = 180°$	$100° + 35° + m\angle D = 180°$
$145° + m\angle C = 180°$	$135° + m\angle D = 180°$
$145° + m\angle C - 145° = 180° - 145°$	$135° + m\angle D - 135° = 180° - 135°$
$m\angle C = 35°$	$m\angle D = 45°$

Because two angles in one triangle are congruent to two angles in the other triangle, the triangles are similar.

Math Talk
Mathematical Practices
Are all right triangles similar? Why or why not?

YOUR TURN

3. **Explain whether the triangles are similar.**



____

____

____

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Finding Missing Measures in Similar Triangles

Because corresponding angles are congruent and corresponding sides are proportional in similar triangles, you can use similar triangles to solve real-world problems.

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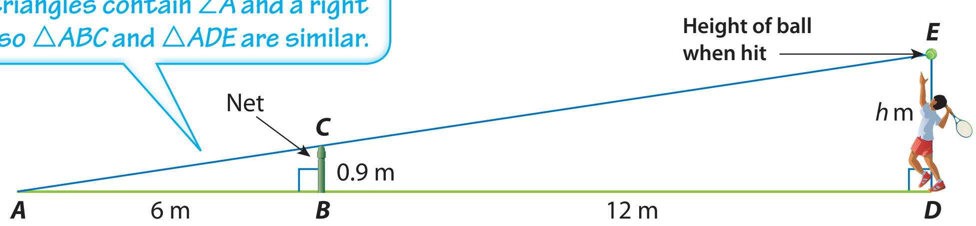
EXAMPLE 2 Real World

FL 8.EE.3.7

While playing tennis, Matt is 12 meters from the net, which is 0.9 meter high. He needs to hit the ball so that it just clears the net and lands 6 meters beyond the base of the net. At what height should Matt hit the tennis ball?

Both triangles contain $\angle A$ and a right angle, so $\triangle ABC$ and $\triangle ADE$ are similar.



In similar triangles, corresponding side lengths are proportional.

$$\frac{AD}{AB} = \frac{DE}{BC} \longrightarrow \frac{6+12}{6} = \frac{h}{0.9}$$ Substitute the lengths from the figure.

$$0.9 \times \frac{18}{6} = \frac{h}{0.9} \times 0.9$$ Use properties of equality to get h by itself.

$$0.9 \times 3 = h$$ Simplify.

$$2.7 = h$$ Multiply.

Matt should hit the ball at a height of 2.7 meters.

Reflect

4. **What If?** Suppose you set up a proportion so that each ratio compares parts of one triangle, as shown below.

$$\begin{array}{rcl} \text{height of } \triangle ABC \longrightarrow & \dfrac{BC}{AB} = \dfrac{DE}{AD} & \longleftarrow \text{height of } \triangle ADE \\ \text{base of } \triangle ABC \longrightarrow & & \longleftarrow \text{base of } \triangle ADE \end{array}$$

Show that this proportion leads to the same value for h as in Example 2.

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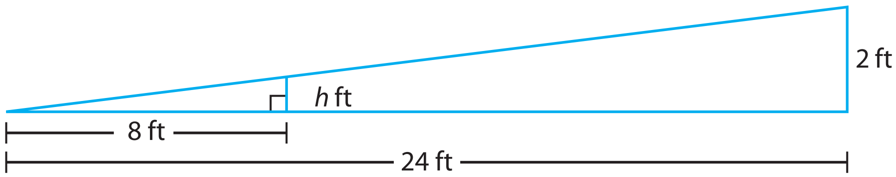
YOUR TURN

5. Rosie is building a wheelchair ramp that is 24 feet long and 2 feet high. She needs to install a vertical support piece 8 feet from the end of the ramp. What is the length of the support piece in inches?



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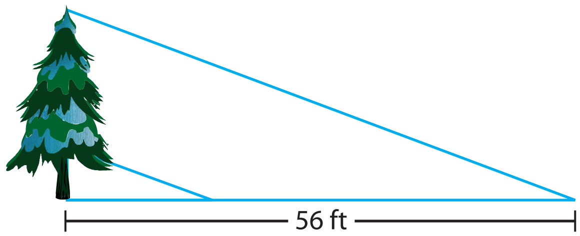
6. The lower cable meets the tree at a height of 6 feet and extends out 16 feet from the base of the tree. If the triangles are similar, how tall is the tree?



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EXPLORE ACTIVITY 2 FL 8.EE.2.6

Using Similar Triangles to Explain Slope

You can use similar triangles to show that the slope of a line is constant.

A Draw a line ℓ that is not a horizontal line. Label four points on the line as *A*, *B*, *C*, and *D*.

You need to show that the slope between points *A* and *B* is the same as the slope between points *C* and *D*.

B Draw the rise and run for the slope between points A and B. Label the intersection as point E. Draw the rise and run for the slope between points C and D. Label the intersection as point F.

C Write expressions for the slope between A and B and between C and D.

Slope between A and B: $\frac{BE}{\square}$ Slope between C and D: $\frac{\square}{CF}$

D Extend $\overleftrightarrow{AE}$ and $\overleftrightarrow{CF}$ across your drawing. $\overleftrightarrow{AE}$ and $\overleftrightarrow{CF}$ are both horizontal lines, so they are parallel.

Line ℓ is a ______________ that intersects parallel lines.

E Complete the following statements:

$\angle BAE$ and ________ are corresponding angles and are ______________.

$\angle BEA$ and ________ are right angles and are ______________.

F By Angle–Angle Similarity, $\triangle ABE$ and ________ are similar triangles.

G Use the fact that the lengths of corresponding sides of similar triangles are proportional to complete the following ratios: $\frac{BE}{DF} = \frac{\square}{CF}$

H Recall that you can also write the proportion so that the ratios compare parts of the same triangle: $\frac{\square}{AE} = \frac{DF}{\square}$.

I The proportion you wrote in step **H** shows that the ratios you wrote in **C** are equal. So, the slope of line ℓ is constant.

Reflect

7. **What If?** Suppose that you label two other points on line ℓ as G and H. Would the slope between these two points be different than the slope you found in the Explore Activity? Explain.

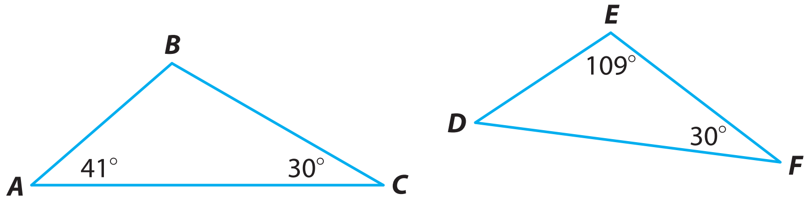
____

____

____

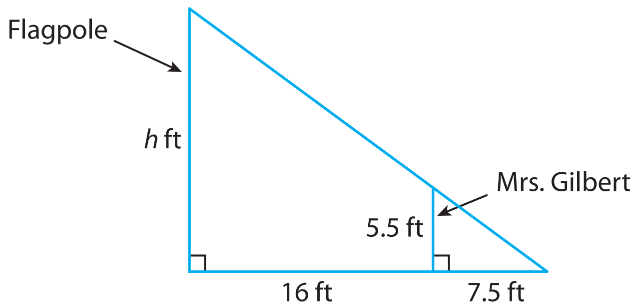
Guided Practice

1. Explain whether the triangles are similar. Label the angle measures in the figure. (Explore Activity 1 and Example 1)



$\triangle ABC$ has angle measures ________________ and $\triangle DEF$ has angle measures ________________. Because ______________ in one triangle are congruent to ____________ in the other triangle, the triangles are ____________.

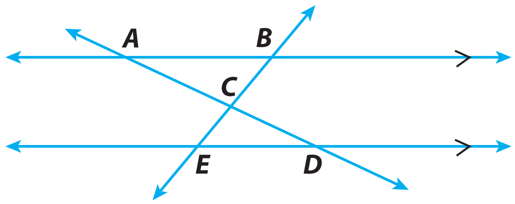
2. A flagpole casts a shadow 23.5 feet long. At the same time of day, Mrs. Gilbert, who is 5.5 feet tall, casts a shadow that is 7.5 feet long. How tall in feet is the flagpole? Round your answer to the nearest tenth. (Example 2)



$$\frac{5.5}{\square} = \frac{h}{\square}$$

$h =$ _____ feet

3. Two transversals intersect two parallel lines as shown. Explain whether $\triangle ABC$ and $\triangle DEC$ are similar. (Example 1)



$\angle BAC$ and $\angle EDC$ are ___________ since they are ___________________________.

$\angle ABC$ and $\angle DEC$ are ___________ since they are ___________________________.

By _____________, $\triangle ABC$ and $\triangle DEC$ are ___________.

ESSENTIAL QUESTION CHECK-IN

4. How can you determine when two triangles are similar?

____

____

____

Name ____________________ Class ____________ Date ____________

11.3 Independent Practice

FL 8.EE.2.6, 8.EE.3.7, 8.G.1.5

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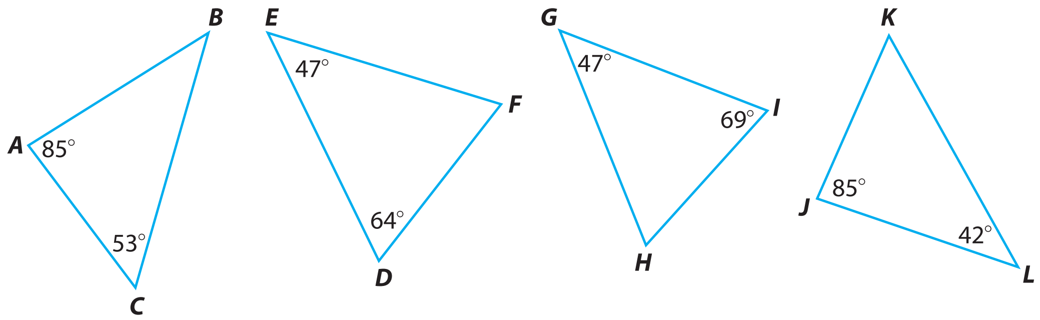
Use the diagrams for Exercises 5–7.



5. Find the missing angle measures in the triangles.

____

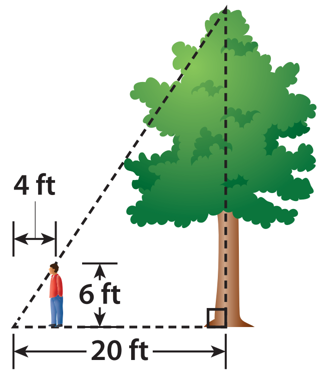
6. Which triangles are similar?

____

7. **Analyze Relationships** Determine which angles are congruent to the angles in $\triangle ABC$.

____

8. **Multistep** A tree casts a shadow that is 20 feet long. Frank is 6 feet tall, and while standing next to the tree he casts a shadow that is 4 feet long.



a. How tall is the tree? ______________

b. How much taller is the tree than Frank? ______________

9. **Represent Real-World Problems** Sheila is climbing on a ladder that is attached against the side of a jungle gym wall. She is 5 feet off the ground and 3 feet from the base of the ladder, which is 15 feet from the wall. Draw a diagram to help you solve the problem. How high up the wall is the top of the ladder?

____

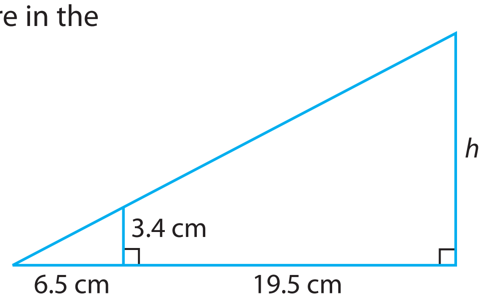
10. **Justify Reasoning** Are two equilateral triangles always similar? Explain.

____

____

____

11. **Critique Reasoning** Ryan calculated the missing measure in the diagram shown. What was his mistake?



$$\frac{3.4}{6.5} = \frac{h}{19.5}$$

$$19.5 \times \frac{3.4}{6.5} = \frac{h}{19.5} \times 19.5$$

$$\frac{66.3}{6.5} = h$$

$$10.2 \text{ cm} = h$$

____

____

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

12. **Communicate Mathematical Ideas** For a pair of triangular earrings, how can you tell if they are similar? How can you tell if they are congruent?

____

____

____

____

13. **Critical Thinking** When does it make sense to use similar triangles to measure the height and length of objects in real life?

____

____

____

14. **Justify Reasoning** Two right triangles on a coordinate plane are similar but not congruent. Each of the legs of both triangles are extended by 1 unit, creating two new right triangles. Are the resulting triangles similar? Explain using an example.

____

____

____

____

____

____

MODULE QUIZ

Ready to Go On?

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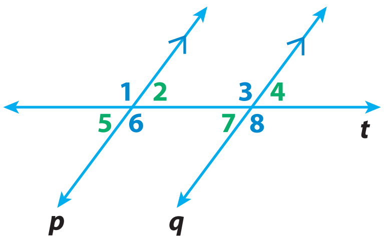
11.1 Parallel Lines Cut by a Transversal

In the figure, line $p \parallel$ line q. Find the measure of each angle if $m\angle 8 = 115°$.



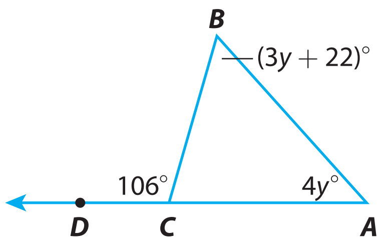
1. $m\angle 7 =$ ____________

2. $m\angle 6 =$ ____________

3. $m\angle 1 =$ ____________

11.2 Angle Theorems for Triangles

Find the measure of each angle.



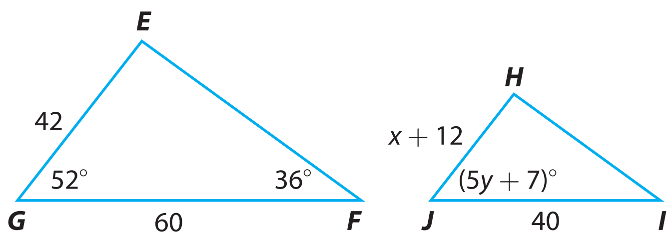
4. $m\angle A =$ ____________

5. $m\angle B =$ ____________

6. $m\angle BCA =$ ____________

11.3 Angle-Angle Similarity

Triangle *FEG* is similar to triangle *IHJ*. Find the missing values.



7. $x =$ ________ **8.** $y =$ ________ **9.** $m\angle H =$ ________

ESSENTIAL QUESTION

10. How can you use similar triangles to solve real-world problems?

____

____

Assessment Readiness

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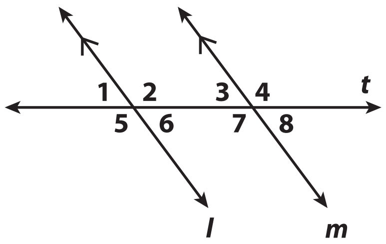
Selected Response

Use the figure for Exercises 1 and 2.



1. Which angle pair is a pair of alternate exterior angles?

Ⓐ $\angle 5$ and $\angle 6$ Ⓒ $\angle 5$ and $\angle 4$

Ⓑ $\angle 6$ and $\angle 7$ Ⓓ $\angle 5$ and $\angle 2$

2. Which of the following angles is **not** congruent to $\angle 3$?

Ⓐ $\angle 1$ Ⓒ $\angle 6$

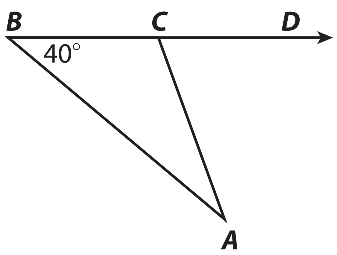
Ⓑ $\angle 2$ Ⓓ $\angle 8$

3. The measures, in degrees, of the three angles of a triangle are given by $2x + 1$, $3x - 3$, and $9x$. What is the measure of the smallest angle?

Ⓐ 13° Ⓒ 36°

Ⓑ 27° Ⓓ 117°

4. Which is a possible measure of $\angle DCA$ in the triangle below?



Ⓐ 36° Ⓒ 40°

Ⓑ 38° Ⓓ 70°

5. Kaylee wrote in her dinosaur report that the Jurassic period was 1.75×10^8 years ago. What is this number written in standard form?

Ⓐ 1,750,000

Ⓑ 17,500,000

Ⓒ 175,000,000

Ⓓ 17,500,000,000

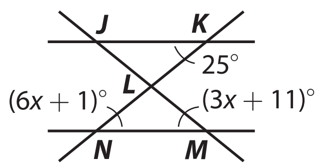
6. Given that y is proportional to x, what linear equation can you write if y is 16 when x is 20?

Ⓐ $y = 20x$ Ⓒ $y = \frac{4}{5}x$

Ⓑ $y = \frac{5}{4}x$ Ⓓ $y = 0.6x$

Mini-Task

7. Two transversals intersect two parallel lines as shown.



a. What is the value of x?

______

b. What is the measure of $\angle LMN$?

______

c. What is the measure of $\angle KLM$?

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d. Which two triangles are similar? How do you know?

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